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STATISTICAL INFERENCE

TABLE 1a: *One Sample*

		TESTS OF SIGNIFICANCE				ESTIMATING WITH CONFIDENCE	
ASSUMPTIONS		**H_0**	**TEST STATISTIC**	**DISTRIBUTION OF TEST STATISTIC UNDER H_0**		**para-meter**	***C* CONFIDENCE INTERVAL**
• Known σ • Normal x <u>or</u> Large n^*		$\mu=\mu_0$	$\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$	N(0,1)		μ	$\bar{x} \mp z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$
• Unknown σ • Normal x		//	$\frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$	t(n-1)		//	$\bar{x} \mp t_{a/2}(n-1)\frac{s}{\sqrt{n}}$
• Unknown σ • Large n^*		//	//	N(0,1)		//	$\bar{x} \mp z_{a/2}\frac{s}{\sqrt{n}}$
• np_0 and $n(1-p_0) \geq 5$ * or • $n\hat{p}$ and $n(1-\hat{p}) \geq 5^*$		$\rho=\rho_0$	$\frac{\hat{\rho}-\rho_0}{\sqrt{\frac{\rho_0(1-\rho_0)}{n}}}$	N(0,1)		ρ	$\hat{p} \mp z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

TABLE 1b: *Two Independent Samples*

ASSUMPTIONS		H_0	TEST STATISTIC	DISTRIBUTION OF TEST STATISTIC UNDER H_0		para-meter	*C* CONFIDENCE INTERVAL
• Known σ_1 and σ_2 • Normal x_1 and x_2 <u>or</u> Large n_1 and n_2^*		μ_1-μ_2	$\dfrac{\bar{x}_1-\bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}}$	N(0,1)		μ_1-μ_2	$\bar{x}_1-\bar{x}_2 \mp z_{a/2}\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}$
• Unknown σ_1 and σ_2 • σ_1=σ_2 • Normal x_1 and x_2		//	$\dfrac{\bar{x}_1-\bar{x}_2}{\sqrt{s_p^2(\dfrac{1}{n_1}+\dfrac{1}{n_2})}}$ $s_p^2=\dfrac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1+n_2-2)}$	$t(n_1+n_2-2)$		//	$\bar{x}_1-\bar{x}_2 \mp t_{a/2}(n_1+n_2-2)$ $\times\sqrt{s_p^2(\dfrac{1}{n_1}+\dfrac{1}{n_2})}$
• Unknown σ_1 and σ_2 • σ_1=σ_2 • Non-normal x_1 and x_2 and Large n_1 and n_2^*		//	//	N(0,1)		//	$\bar{x}_1-\bar{x}_2 \mp z_{a/2}\sqrt{s_p^2(\dfrac{1}{n_1}+\dfrac{1}{n_2})}$
• Unknown σ_1 and σ_2 • σ_1≠σ_2 • Normal x_1 and x_2		//	$\dfrac{\bar{x}_1-\bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}}}$	*t(df)* where $df=\dfrac{(\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2})^2}{\dfrac{1}{n_1-1}\left(\dfrac{s_1^2}{n_1}\right)^2+\dfrac{1}{n_2-1}\left(\dfrac{s_2^2}{n_2}\right)^2}$ if n_1 and $n_2 \geq 5$, otherwise use $df=\min\{n_1-1,\ n_2-1\}$		//	$\bar{x}_1-\bar{x}_2 \mp t_{a/2}(df)$ $\times\sqrt{\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}}$

TABLE 1b: *Two Independent Samples* (it continues from page 2)

ASSUMPTIONS		$\mathbf{H_0}$	**TEST STATISTIC**	**DISTRIBUTION OF TEST STATISTIC UNDER $\mathbf{H_0}$**		**para-meter**	***C* CONFIDENCE INTERVAL**
$n_1 \times \min\{\hat{p}, 1-\hat{p}\} \geq 5^*$, and $n_2 \times \min\{\hat{p}, 1-\hat{p}\} \geq 5^*$ where $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$		$p_1 = p_2$	$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$, $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$	N(0,1)		$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2 \mp z_{a/2} \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$
• Normal x_1 and x_2		$\sigma_1^2 = \sigma_2^2$	$\mathrm{F} = \frac{\text{larger s}^2}{\text{smaller s}^2}$	$F(n_1 - 1,\ n_2 - 1)$ if $s_1^2 > s_2^2$ or $F(n_2 - 1,\ n_1 - 1)$ if $s_1^2 < s_2^2$			

TABLE 1c: *Two Paired Samples*

• Normal x_1 and x_2		$\mu_1 = \mu_2$	$\frac{\bar{d}}{s_d / \sqrt{n}}$, $d_i = x_{1i} - x_{2i},\ \ i = 1,2,\cdots n$	t(n -1)		$\mu_1 - \mu_2$	$\bar{d} \mp t_{a/2}(n-1)\frac{s_d}{\sqrt{n}}$
• Non-normal x_1 and x_2 and Large n*		//	//	N(0,1)		//	$\bar{d} \mp z_{a/2}\frac{s_d}{\sqrt{n}}$

* An asterisk indicates approximate tests and confidence intervals.